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PROCESS DESIGN AND CONTROL

A Hierarchical Lagrangean Relaxation Procedure for Solving Midterm Planning Problems

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An efficient decomposition procedure for solving midterm planning problems is developed based on Lagrangean relaxation. The basic idea of the proposed solution technique is the successive partitioning of the original problem into smaller, more computationally tractable subproblems by hierarchical relaxation of key complicating constraints. The systematic identification of these complicating constraints is accomplished by utilizing linear programming relaxation dual-multiplier information. This hierarchical Lagrangean relaxation procedure, along with an upper bound generating heuristic, is incorporated within a subgradient optimization framework. This solution strategy is found to be much more effective, in terms of both quality of solution and computational requirements, than commercial mixed-integer linear programming solvers in bracketing the optimal value, especially for larger problems.

1. Introduction

In today's ever-changing markets, maintaining an efficient and flexible supply chain is critical for every enterprise. In order to retain and strengthen their competitive edge in the market, organizations need to coordinate and integrate all their business operations right from the production stage to the distribution stage. Systematic planning and scheduling techniques based on mathematical programming principles have been developed to address these vital issues of supply-chain management.

Most of the production planning problems can be viewed as extensions of the classic economic lot-sizing problem.[1] This problem involves determining the production levels of multiple products which have deterministic demands due at the end of a finite number of planning time periods. A fixed setup cost is incurred along with a constant unit production cost and an inventory holding cost in each time period. The key constraints of the problem are the inventory balance constraint and the capacity utilization constraint. The objective of the problem is to satisfy the total demand at a minimum cost with the available capacity. The problem described above is more specifically referred to as the multi-item capacitated lot-sizing problem (MICLSP). The key trade-off involved is between fixed setup and inventory holding costs as high (low) production levels lead to low (high) setup costs and high (low) inventory costs. Balancing these two cost components is the main objective of the lot-sizing problem. The computational complexity of this mixed-integer programming (MIP) problem has been extensively studied. The single-item uncapacitated version of the MICLSP can be efficiently solved with the Wagner−Whitin dynamic programming algorithm.[2] However, the capacitated problem has been shown to be NP-hard even for the single-item case.[3,4] The resource-constrained multi-item formulation, which describes most practical planning and scheduling problems, is even harder to solve. Aside from being NP-hard, it is also computationally intractable in the sense that it is difficult to obtain "good", not necessarily optimal, solutions for large-scale problems in reasonable computation time. Various solution techniques have been explored in order to get optimal or good suboptimal solutions for the somewhat idealized variations of the lot-sizing problem. These include cut generation techniques,[5,6] variable redefinition,[7] Benders decomposition,[8,9] cross decomposition,[10] Lagrangean relaxation,[11−16] and heuristic search techniques.[17,18]

The planning models available in the process systems engineering literature can be broadly categorized into three distinct groups based on the time frames they address. Long-range planning or capacity expansion models[19−25] are "strategic" planning models which aim to identify the optimal timing, location, and extent of additional investments in processing networks over a relatively long time horizon. Short-term scheduling models[26−33] or "operational" planning models constitute the other extreme of the spectrum of planning models. These models are characterized by very short time periods over which the various manufacturing tasks have to be fully sequenced. The third class of models, the midterm planning models,[34,35] is intermediate in nature. These "tactical" planning models, closest in structure to the MICLSP, incorporate some features from both the long-term and the short-term models. For

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example, the midterm planning formulation of McDonald and Karimi,[35] which is going to serve as the benchmark formulation in this work, models the inventory balance much like the short-term scheduling models. Similarly, contrary to most short-term scheduling models which deal primarily with a single manufacturing site, the midterm formulation is characterized by multiple production facilities. This overlap and consolidation of modeling features in the midterm planning problem make it a challenging problem to solve.

Most of the work in chemical engineering literature has dealt primarily with either the capacity expansion problem or the short-term scheduling problem. A number of specially tailored solution techniques have been developed for efficiently solving large-scale instances of these problems. Cutting plane techniques and Benders decomposition are discussed in Sahinidis et al.[19] for the long-range planning problem. Sahinidis and Grossmann[22] give a strong reformulation of the capacity expansion problem which is based on variable disaggregation. This results in a large number of additional variables which are projected out using polyhedral theory in Liu and Sahinidis.[36] Iyer and Grossmann[25] developed a bilevel decomposition algorithm which solves the long-range planning problem in the reduced space of the binary variables. For short-term scheduling problems, a number of time-based decomposition approaches are presented in Bassett et al.[37] Schilling and Pantelides[38] described a RTN-type formulation for short-term scheduling. Lately, Ierapetritou and Floudas[39,40] discussed novel mathematical formulations based on a continuous-time representation for the short-term scheduling of batch as well as continuous and semicontinuous processes. While extensive research work exists for short-term and long-term planning formulations, very little attention has been devoted at devising customized solution procedures for the computationally challenging midterm planning formulations. In this paper, a decomposition strategy is explored which successively partitions the original problem into smaller, more tractable subproblems. This decomposition is driven by a hierarchical Lagrangean relaxation-based procedure aimed at providing tight lower bounds to the original problem. The information contained in the Lagrangrean relaxation solutions provides the basis for generating upper bounds by a heuristic procedure. This lower and upper bounding scheme is implemented within an iterative framework and customized for the efficient solution of the midterm planning model of McDonald and Karimi.[35]

This paper is organized as follows: In the next section the midterm planning formulation of McDonald and Karimi[35] is briefly discussed. Next, a brief introduction to the Lagrangean relaxation technique is provided followed by the proposed hierarchical Lagrangean relaxation (HLR) procedure. Applications of the proposed solution algorithm to two example supply chains are then presented followed by concluding remarks.

2. Midterm Production Planning Problem

The midterm production planning model of McDonald and Karimi[35] is adopted in this work to serve as a benchmark for the proposed decomposition algorithm. Nevertheless, the developed procedure can be applied with only minor modifications to general planning formulations having the underlying structure of the MICLSP problem. This formulation,[35] which has the structure of a multi-item, multi-facility capacitated lot-sizing problem, balances costs incurred in the supply chain subject to production and supply-chain constraints. The production facilities process multiple products on one or more semicontinuous single-stage processors. Demands for these products are imposed by external customers at the end of each time period of the planning horizon. The optimal operating policy for the manufacturing facilities is determined so that these demands can be met effectively. Other activities, such as inventory management, are also coordinated over the entire enterprise to keep the total cost at a minimum. The midterm production planning model provides an elaborate description of the complex supply chain of large chemical companies. The formulation is flexible as additional features such as customer qualification and transportation time lags can be easily incorporated. The planning horizon for this model, which is typically between 1 and 3 years, lies in between the planning horizons for the long-term planning models (5−10 years) and the short-term scheduling models (2−6 months). This is divided into time periods of about 1 month duration.

The following notation is used in the model formulation:[35]

Sets

$I = \{i\}$ = set of products
$I^{RM} \subset I = \{i\}$ = set of raw materials
$I^{IP} \subset I = \{i\}$ = set of intermediate products
$I^{FP} \subset I = \{i\}$ = set of finished products
$F = \{f\}$ = set of product families
$J = \{j\}$ = set of machines
$S = \{s\}$ = set of facilities where these machines are located
$\mathcal{T} = \{t\}$ = set of time periods
$C = \{c\}$ = set of customers

Parameters

h_{ist} = inventory holding cost for a unit of product i at site s for the duration of time period t
μ_{ic} = revenue per unit of product $i \in I^{FP}$ sold to customer c
p_{is} = price of raw material $i \in I^{RM}$ at site s
ζ_{is} = penalty for dipping below safety target of product i at site s
ν_{ijs} = variable cost for producing a unit of product $i \in I \setminus I^{RM}$ on processor j at site s
R_{ijst} = rate of production of product $i \in I \setminus I^{RM}$ on processor j at site s in time period t
$\beta_{i'is}$ = yield adjusted amount of raw or intermediate $i \in I \setminus I^{FP}$ that must be consumed to produce a unit of $i' \in I \setminus I^{RM}$ at site s
H_{jst} = amount of time available for production on processor j at site s in time period t
MRL_{fjs} = minimum run length for family f on processor j at site s
FC_{fjs} = fixed cost of production for family f on processor j at site s
τ_{if} = 0−1 parameter indicating whether product i belongs to family f
d_{ict} = demand for finished product $i \in I^{FP}$ at customer c due at the end of time period t
I^{L}_{ist} = safety stock target for product i at site s in time period t
I^{0}_{is} = inventory of product i at site s at the start of the planning horizon
$t_{sc}/t_{ss'}$ = transportation cost to move a unit of product from site s to site s' or customer c

Variables

$$Y_{fjst} = \begin{cases} 1 & \text{if family } f \text{ is processed on machine } j \\ & \text{at site } s \text{ in time period } t \\ 0 & \text{otherwise} \end{cases}$$

P_{ijst} = production amount of $i \in I \setminus I^{RM}$ on processor j at site s in time period t

RL_{ijst} = corresponding run length of product $i \in I \setminus I^{RM}$ on processor j at site s in time period t

FRL_{fjst} = run length for family f on processor j at site s in time period t

C_{ist} = consumption of raw material or intermediate $i \in I \setminus I^{FP}$ at site s in time period t

$\sigma_{iss't}$ = flow of intermediate product $i \in I^{IP}$ from facility s to s' in time period t

S_{isct} = supply of finished product $i \in I^{FP}$ from facility s to customer c in time period t

I_{ist} = inventory level for $i \in I^{FP}$ at the end of time period t at site s

I^{-}_{ict} = amount of shortage of finished product $i \in I^{FP}$ at customer c in time period t

I^{Δ}_{ist} = deviation below safety stock target for product $i \in I$ at site s in time period t

Using the notation listed above, the midterm planning model of McDonald and Karimi[35] is formulated as the following mixed-integer linear problem:

$$\text{(MP):} \quad \min z = \sum_{f,j,s,t} FC_{fjs} Y_{fjst} + \sum_{i,j,s,t} \nu_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,s',t} t_{ss'} \sigma_{iss't} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,t} \zeta_{is} I^{\Delta}_{ist} + \sum_{i,c,t} \mu_{ic} I^{-}_{ict}$$

subject to

$$P_{ijst} = R_{ijst} RL_{ijst} \qquad \forall\, i \in I \setminus I^{RM}, j, s, t \qquad (1)$$

$$C_{ist} = \sum_{i':\beta_{i'is} \neq 0} \beta_{i'is} \sum_j P_{i'jst} \qquad \forall\, i \in I \setminus I^{FP}, s, t \qquad (2)$$

$$C_{ist} = \sum_{s'} \sigma_{is'st} \qquad \forall\, i \in I^{IP}, s, t \qquad (3)$$

$$P_{ijst} \leq R_{ijst} H_{jst} \qquad \forall\, i, j, s, t \qquad (4)$$

$$RL_{ijst} \leq H_{jst} \qquad \forall\, i, j, s, t \qquad (5)$$

$$FRL_{fjst} = \sum_{i \in \tau_{if}} RL_{ijst} \qquad \forall\, f, j, s, t \qquad (6)$$

$$\sum_f FRL_{fjst} \leq H_{jst} \qquad \forall\, j, s, t \qquad (7)$$

$$FRL_{fjst} - H_{jst} Y_{fjst} \leq 0 \qquad \forall\, f, j, s, t \qquad (8)$$

$$FRL_{fjst} - MRL_{fjs} Y_{fjst} \geq 0 \qquad \forall\, f, j, s, t \qquad (9)$$

$$I_{ist} = I_{is(t-1)} + \sum_j P_{ijst} - \sum_{s'} \sigma_{iss't} - \sum_c S_{isct} \qquad \forall\, i \in I \setminus I^{RM}, s, t \qquad (10)$$

$$I^{-}_{ict} \geq I^{-}_{ic(t-1)} + d_{ict} - \sum_s S_{isct} \qquad \forall\, i \in I^{FP}, c, t \qquad (11)$$

$$\sum_{s,t' \leq t} S_{isct'} \leq \sum_{t' \leq t} d_{ict'} \qquad \forall\, i, c, t \qquad (12)$$

$$I^{\Delta}_{ist} \geq I^{L}_{ist} - I_{ist} \qquad \forall\, i, s, t \qquad (13)$$

$$S_{isct} \leq \sum_{t' \leq t} d_{ict'} \qquad \forall\, i, s, c, t \qquad (14)$$

$$I^{-}_{ict} \leq \sum_{t' \leq t} d_{ict'} \qquad \forall\, i, s, c, t \qquad (15)$$

$$I^{\Delta}_{ist} \leq I^{L}_{ist} \qquad \forall\, i, s, t \qquad (16)$$

$$P_{ijst}, RL_{ijst}, FRL_{fjst}, C_{ist}, \sigma_{iss't}, I_{ist}, I^{-}_{ict}, S_{isct}, I^{\Delta}_{ist} \geq 0$$

$$Y_{fjst} \in \{0, 1\}$$

The objective function of formulation MP minimizes the sum of manufacturing and supply-chain costs incurred in the production−distribution system of a typical process industry. These costs consist of the raw material costs, variable and fixed production costs, inventory holding, transportation, and underproduction charges. This objective function is minimized subject to the various constraints defining the system. These constraints are briefly discussed below. Equation 1 defines the production amount in terms of the rate of production and the campaign run length. The bill of materials relations are utilized in eq 2 to model the consumption of intermediate products. Equation 3 eliminates redundant flows in the supply chain by ensuring that products shipped to a particular site in a given time period are consumed in the same time period. Equations 4 and 5 provide upper bounds on production and run lengths, respectively. Equation 6 is the family run length defining constraint. Equation 7 is the capacity competition constraint which ensures that the total production time does not exceed the time available for production. Equations 8 and 9 are the setup enforcing constraints. These constraints ensure that a fixed cost is incurred whenever there is a production run. Equation 10 represents the basic inventory balance at the end of a time period. Equation 11 indicates the carryover of customer shortage from one period to the next. Equation 12 safeguards against oversupply. Forcing the inventory shortfall to the maximum of zero and the deviation below the safety stock target is achieved by eq 13 in conjunction with the non-negativity of the inventory shortfall variable. Equations 14−16 provide upper bounds for the supply, customer shortage, and inventory shortfall, respectively.

The planning problem constraints can be divided into two distinct categories: *Manufacturing* (or production) constraints (eqs 1−9) establish the capacity utilization policy as well as the consumption and production of products at the manufacturing facilities. The second group of constraints (eqs 10−16) is denoted as the *supply-chain constraints*. These constraints link the

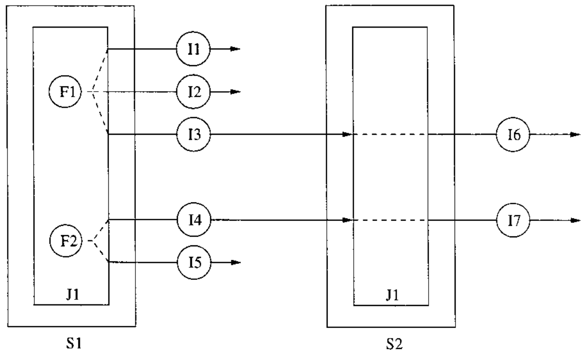

Figure 1. Simplified supply chain of a typical process industry.

supply of products to the customer with the production site inventory. The idea of distinguishing between production constraints and supply-chain constraints is extended to the decision variables. Variables RL_{ijst}, FRL_{fjst}, P_{ijst}, C_{ist}, $\sigma_{iss't}$, and Y_{fjst} are referred to as *production variables*. They can be construed as decision variables that determine the production levels at different manufacturing facilities. The remaining variables, S_{isct}, I_{ist}, I^{Δ}_{ist}, and I^{-}_{ict}, comprise the *supply-chain variables* establishing the product supply and inventory policy. The midterm planning formulation thus tries to obtain an optimal allocation of an enterprise's resources over the production and supply-chain processes.

A simplified pictorial representation of the supply-chain modeled by formulation MP is shown in Figure 1. The production system shown comprises of two sites S_1 and S_2, each having a single processor J_1. Products I_1-I_7 are produced at these two facilities. Demands for each of these products may exist for one or more customers. Products at site S_1 have been aggregated into two product families. Family F_1 comprises of products I_1, I_2, and I_3 while products I_4 and I_5 constitute family F_2. The concept of a family is employed to lump products with identical "routes" through the supply-chain network. At the expense of enforcing the same production schedule for all products belonging to a family, a significant reduction in the combinatorial alternatives is achieved.[35] Finally, products I_3 and I_4 serve as intermediates in site S_2 for producing products I_6 and I_7.

Formulation MP utilizes the concept of slots for modeling the planning horizon. The exact sequencing and timing of processing events is thus not considered. This implies that run lengths are assumed to be much smaller than the slot lengths. This prevents the "spill-over" of a production run from one time period to the next. This assumption can, however, be relaxed by introducing additional binary variables into the formulation, as discussed by McDonald and Karimi.[35] In the spirit of the classic MICLSP, the key trade-off involved in the midterm planning problem is between inventory holding costs and setup costs.

Problem MP is a difficult problem because of the presence of binary variables modeling (i) the fixed-charge cost term incurred for each setup and (ii) the minimum run length constraint. Both of these modeling features attempt to restrict the number of setups and the problem remains difficult after the omission of one of the two. Direct solution of problem MP using commercial solvers such as OSL or CPLEX first produces good solutions with relatively small relaxation gaps. Subsequent iterations, though, fail to substantially reduce this gap. This result is consistent with the fact that the underlying structure of formulation MP is the classic MICLSP, which is NP-hard.[3]

In the context of this paper, we explored a number of solution approaches with an emphasis on decomposition procedures. Benders decomposition,[41] which is one of the standard decomposition techniques, was thoroughly explored. This technique failed to provide tight lower bounds for the midterm planning problem even after a large number of iterations (see examples section). On the other hand, as will be discussed in the next section, Lagrangean relaxation involving a judicious selection of the complicating constraints provides encouraging results. An attempt to combine Benders decomposition and Lagrangean relaxation through cross decomposition[42] was unsuccessful because of the low quality of the Benders cuts. Therefore, a hierarchical decomposition through Lagrangean relaxation is pursued in this paper, exploiting both the primal and dual structure of the underlying MICLSP model.

3. Proposed Solution Procedure

The basic idea of the proposed solution methodology is to bracket the optimal solution by decomposing the original problem into a collection of smaller, more tractable subproblems. A three-stage hierarchical Lagrangean relaxation scheme drives the decomposition procedure, providing lower bounds to the original problem. Upper bounds are obtained by utilizing the information obtained from the Lagrangean relaxations. A key issue which determines the quality of the problem relaxations is the inherent trade-off between the quality of lower bounds obtained and the computational effort involved in solving the relaxed problem. Specifically, relaxing a large number of constraints may result in easy to solve subproblems, but the obtained lower bound will most likely be poor. On the other hand, if only a few constraints are relaxed, aiming at a tight lower bound, the resulting subproblems may be intractable. A hierarchical three-stage decomposition scheme aimed at setting the optimal trade-off between the degree of relaxation and solvability is introduced in this paper. The employed relaxation method (i.e., Lagrangean) is briefly discussed in the next subsection.

3.1. Overview of Lagrangean Relaxation. Lagrangean relaxation provides an elegant way for obtaining lower bounds for certain classes of combinatorial problems. These problems are characterized by a set of side (i.e., complicating) constraints whose removal yields a minimization problem which either is decomposable over subsets of variables or has special structure (e.g., knapsack problem) that can be exploited by customized algorithms. In Lagrangean relaxation, the complicating constraints are removed from the constraint set and replaced with a penalty term in the objective function.[43-45] Consider the following problem:

$$(\mathrm{P}): \qquad \min \{c^{\mathrm{T}}x | Ax \le b,\ Cx \le d,\ x \in X\} \tag{17}$$

The Lagrangean relaxation of P relative to the complicating constraint $Ax \le b$ is defined as

$$\mathrm{LR}_{\lambda} = \min \{c^{\mathrm{T}}x + \lambda^{\mathrm{T}}(Ax - b) | Cx \le d,\ x \in X\} \tag{18}$$

The problem

$$\mathrm{LR} = \max_{\lambda \ge 0} \mathrm{LR}_{\lambda} \tag{19}$$

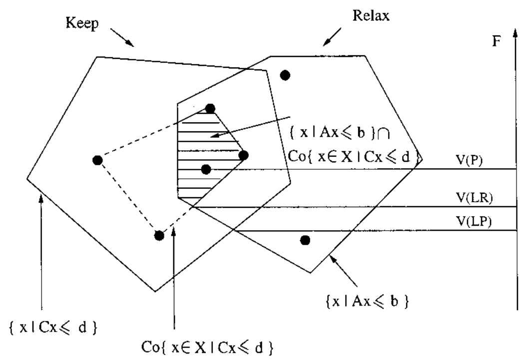

Figure 2. Geometric interpretation of Lagrangean relaxation.

is called the *Lagrangean dual* relative to constraint $Ax \le b$, which is referred to as the *dualized constraint set*, and λ is the Lagrange multiplier or dual vector. The dualized constraint set must be appropriately chosen so that problem LR_λ is easy to solve. Note that since $\lambda \ge 0$ and $Ax - b \le 0$ for every optimal solution x of P we have

$$V(LR_\lambda) \le V(LR) \le V(P)$$

where $V(\cdot)$ is the optimal solution operator. Furthermore, it can be shown that the Lagrangean dual LR is equivalent with the original problem after replacing the noncomplicating constraints with their convex hulls:[45]

$$(P^*): \quad \min \{c^T x | Ax \le b, \mathrm{Co}\{Cx \le d, x \in X\}\} \tag{20}$$

where $\mathrm{Co}(\cdot)$ is the convex hull operator. Therefore,

$$V(LP) \le V(P^*) = V(LR) \le V(P)$$

where $V(LP)$ is the optimal LP relaxation value. In addition, if the optimal LP relaxation multipliers for the complicating constraints are used to derive $V(LR_\lambda)$, then it can be shown that the bound obtained is at least as good as the LP relaxation bound.

$$V(LR_\lambda) \ge V(LP)$$

If $\mathrm{Co}\{Cx \le d, x \in X\} = \{Cx \le d, x \in X\}$, then $V(P) \ge V(P^*) = V(LR) = V(LP)$. In this case, the Lagrangean dual bound provides no improvement over the LP relaxation bound. However, if $\mathrm{Co}\{Cx \le d, x \in X\} \subset \{Cx \le d, x \in X\}$, then the Lagrangean dual bound is strictly better than the LP bound. Clearly, complying with the latter relation determines the selection process of the complicating constraints. Figure 2 (see Guignard[46]) provides a geometric interpretation of Lagrangean relaxation.

Subgradient optimization is typically employed to maximize $V(LR_\lambda)$ over λ. This involves searching the dual variable space starting from the LP relaxation Lagrange multiplier values using the following updating procedure:

$$\lambda^{(l+1)} = \max\left\{0, \lambda^{(l)} + \frac{a^{(l)}(UBD - LR_{\lambda^{(l)}})(Ax - b)}{||Ax - b||^2}\right\} \tag{21}$$

Here $a^{(l)}$ is a scalar step size, whose value is monotonically decreasing. Convergence to the optimum is asymptotic. Excellent reviews on Lagrangean relaxation can be found by Visweswaran[47] and Guignard.[46]

3.2. HLR Procedure. It is evident from the above discussion that selecting which constraints to dualize (complicating constraints) is, in general, a nontrivial task. For example, the inventory balance constraint (eq 10) appears to be an intuitive choice to relax as it is the only constraint that couples production and supply-chain variables. The Lagrangean relaxation of the inventory balance constraint decomposes the original problem into a production setting subproblem and a supply-chain management subproblem. However, it is found that the LR bound obtained in this case never improves over the LP relaxation bound. This is consistent with the results of Chen and Thizy[48] for the MICLSP model. Therefore, an alternative relaxation scheme has to be explored to obtain bounds tighter than the LP relaxation. In the context of the MICLSP formulation, relaxation of the capacity constraint has been suggested in the OR literature.[49] In fact, Bitran and Matsuo[49] have shown that the gap between the above-mentioned relaxation and the original problem decreases as the total number of products increases. This relaxation decouples formulation MICLSP over single products, yielding a set of separable single-item multi-site problems.[13,14]

3.2.1. Stage 1. The relaxation of the capacity competition constraint in the midterm planning problem MP (eq 7) defines the first stage of the proposed hierarchical relaxation procedure. The relaxation of this constraint set provides the tightest lower bounds, and thus it is moved at the top of the hierarchical relaxation. The capacity competition constraint ensures that the run lengths of all products on a particular machine at a given site in a given time period do not exceed the total time available for production. When multipliers λ_{jst} are assigned to each capacity competition constraint and dualized, the following problem is obtained:

$$(LRMP): \quad \min z = \sum_{f,j,s,t} FC_{fjs} Y_{fjst} + \sum_{i,j,s,t} \nu_{ijs} P_{ijst} + \sum_{i,s,t} p_{is} C_{ist} + \sum_{i,s,t} h_{ist} I_{ist} + \sum_{i,s,s',t} t_{ss'} \sigma_{iss't} + \sum_{i,s,c,t} t_{sc} S_{isct} + \sum_{i,s,t} \zeta_{is} I^{\Delta}_{ist} + \sum_{i,c,t} \mu_{ic} I^{-}_{ict} + \sum_{j,s,t} \lambda_{jst} (\sum_f FRL_{fjst} - H_{jst})$$

subject to

constraints 1−6 and 8−16

$$P_{ijst}, RL_{ijst}, FRL_{fjst}, C_{ist}, \sigma_{iss't}, S_{isct}, I_{ist}, I^{\Delta}_{ist}, I^{-}_{ict} \ge 0$$

$$Y_{fjst} \in \{0, 1\}$$

While for the standard MICLSP model stage 1 relaxation completely decouples the original problem into single-item subproblems, this is not the case for the more complex MP formulation. In the resulting relaxation LRMP of MP, some product linking remains through the bill of materials (BOM) or material balance constraint (eq 2) which establishes the mass balances between raw materials and products. Thus, decomposition over subgroups of products rather than over single products is achieved. This is illustrated with the help of the supply chain shown in Figure 1. If the capacity competition constraints at both sites S_1 and S_2 are relaxed, then the problem decomposes into subproblems $SP_1^{st=1}$ and $SP_2^{st=1}$, as shown in Figure 3. Problem $SP_1^{st=1}$ involves products I_1, I_2, I_3, and I_6 while $SP_2^{st=1}$ involves

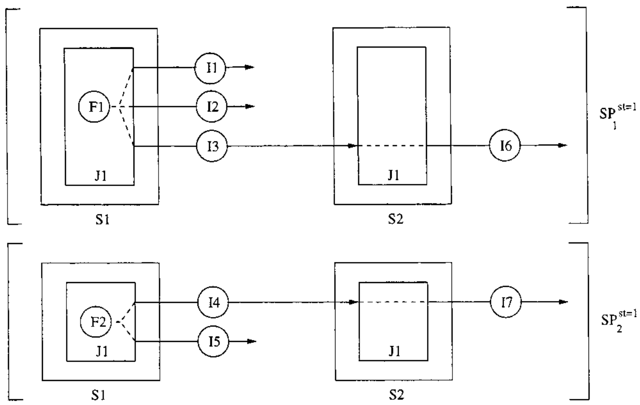

Figure 3. Horizontal partitioning by first-stage relaxation.

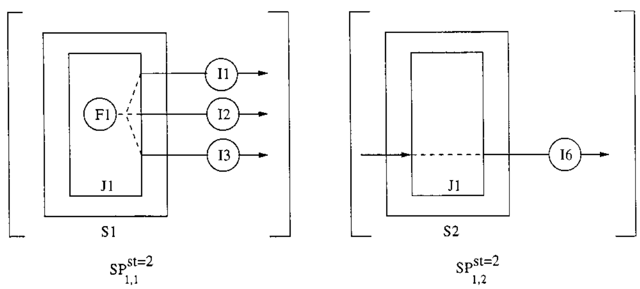

Figure 4. Vertical partitioning by second-stage relaxation.

products I_4, I_5, and I_7. In general, problem LRMP is composed of *m* first-stage separable subproblems, denoted as $SP^{st=1}_m$, over disjoint subsets of products which are not linked through the BOM constraints. This first stage of problem decomposition is referred to as *horizontal partitioning*.

The first-stage subproblems are more tractable than the original problem, but they are still NP-hard. Bitran and Yanasse[3] have shown that even special cases of single-item lot-sizing problem solvable in polynomial time become NP-hard after the introduction of a second item. This implies that some of the first-stage subproblems may potentially be computationally intractable and hence would require further decomposition. The work of Bitran and Yanasse[3] suggests that further decomposition over production sites is a promising alternative. This defines the second stage of relaxation.

3.2.2. Stage 2. Stage 2 relaxation involves relaxation of the BOM constraints (eq 2) which establish links between raw materials and products. Conceptually, this decouples different production sites in the supply chain, and hence it is referred to as *vertical partitioning*. Relaxation of the BOM constraints further decomposes the first-stage subproblems over production sites. For example, suppose that subproblem $SP^{st=1}_1$ (Figure 3) is intractable. Relaxation of the BOM constraint linking products I_3 and I_6 results in two second-stage subproblems, one over site S_1 and the other over S_2, as shown in Figure 4. In general, the stage 2 relaxation decomposes any intractable stage 1 subproblem $SP^{st=1}_m$ into *n* second-stage subproblems $SP^{st=2}_{m,n}$ by relaxing some or all of the BOM constraints. A site is referred to as either *capacitated* or *uncapacitated* depending on whether an upper bound H_{jst} is enforced on the run lengths RL_{ijst}. Computational experience indicates that it is efficient to keep at least one capacitated site for each second-stage subproblem, restricting second-stage relaxation over only capacitated sites. This prevents isolation of uncapacitated sites, which tend to provide poor lower bounds. This vertical partitioning of the supply chain constitutes the second phase of the hierarchical relaxation scheme.

A key issue associated with the second-stage relaxation is how many and which BOM constraints need to be relaxed. This is critical as it governs the strength of lower bounds obtained and the computational tractability of the second-stage subproblems. Obviously, the relaxation of the smallest possible number of BOM constraints yielding tractable second-stage subproblems is desired. For example, for three consecutive sites, S_1, S_2, and S_3, the question is whether it is better to first decompose between sites S_1 and S_2 or between sites S_2 and S_3. To answer this question, a procedure is introduced which uses the information provided by the LP relaxation Lagrange multipliers associated with the BOM constraints. This procedure is explained with respect to a general LP problem.

Consider the following LP in standard form:

$$z = \min c^T x$$
$$A_1 x \geq b_1 \leftarrow \lambda_1 \geq 0$$
$$A_2 x \geq b_2 \leftarrow \lambda_2 \geq 0$$
$$x \geq 0$$

Let λ_1 and λ_2 be the Lagrange (dual) multipliers associated with the constraints. Relaxation of a particular constraint, for example, $A_1x \geq b_1$, can be achieved by simply omitting it from the constraint set. Alternately, this can also be viewed as decreasing the right-hand side b_1 of the constraint to a sufficiently small value to ensure that this constraint is redundant. From LP duality theory it is known that

$$\lambda_1 = \left(\frac{\partial z}{\partial b_1}\right)_{b_2} \quad \text{and} \quad \lambda_2 = \left(\frac{\partial z}{\partial b_2}\right)_{b_1}$$

Thus, for a unit decrease in the value of the constant term b_1, the Lagrange multiplier λ_1 equals the corresponding decrease in the objective function. This implies that a smaller relaxation gap (based on local information) will be obtained if the constraint with the smaller Lagrange multiplier is relaxed. Therefore, if $\lambda_1 \leq \lambda_2$, then the first set of constraints is more likely to provide a tighter relaxation.

Going back to the original problem, relaxation of only the constraint with the smallest Lagrange multiplier associated with the BOM constraints is first employed. If a single site-to-site disconnection retains some intractable second-stage subproblems, then the BOM constraint with the second smallest Lagrange multiplier associated with these subproblems is relaxed. This is continued until all subproblems are tractable or, otherwise, all capacitated sites have been decoupled. The intractable stage 2 subproblems, each containing a single capacitated site, are then subjected to stage 3 relaxation.

3.2.3. Stage 3. The third-stage relaxation addresses the solution of single-capacitated site problems which remain intractable with a commercial MILP solver (e.g., CPLEX 4.0) within the alloted time (e.g., 10−20 s). The NP-hard characterization of these subproblems is due to the presence of the capacity constraint (eq 8). Relaxation of this constraint yields uncapacitated,

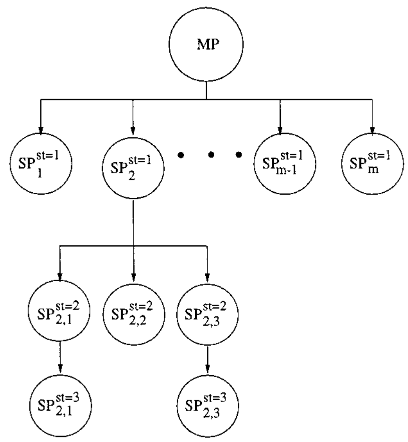

Figure 5. Decomposition structure obtained by the three-stage HLR procedure.

single-product, single-site problems $SP_{m,n}^{st=3}$ which can be solved in polynomial time.[3] This defines the third and last stage in the hierarchy of relaxations because the resulting subproblems are no longer NP-hard. The LP relaxation Lagrange multiplier based constraint identification procedure is employed to decide which capacity constraints to relax. Unlike stages 1 and 2, which impose horizontal and vertical cuts on the supply chain, respectively, stage 3 does not decouple the supply chain any further but rather changes the processing attributes of the sites from capacitated to uncapacitated.

Figure 5 summarizes the proposed three-stage HLR procedure. This procedure progressively partitions the original problem into smaller subproblems, as shown in Figure 5. Specifically, the original problem MP is decomposed into M ($m = 1, ..., M$) first-stage subproblems $SP_m^{st=1}$ by relaxing the capacity competition constraints. Some of these subproblems may solve to optimality in the alloted time. These tractable first-stage subproblems are those which do not have any successor nodes in Figure 5. The first-stage subproblems which do not solve in the specified resource limit are further decomposed into subproblems $SP_{m,n}^{st=2}$ ($n = 1, ..., N_m$) by relaxing the BOM constraints. Relaxation of the capacity constraints is required only for those stage 2 subproblems which fail to solve in the specified computational time.

The sum of the optimal solutions for the first-stage subproblems $SP_m^{st=1}$ ($m = 1, ..., M$) provides a valid bound for the optimal solution of the original problem MP. This is because the first-stage subproblems are obtained by relaxing (dualizing) the capacity competition constraint in the MP formulation, yielding a set of separable lower bounding problems. If the optimal solution of one of the first-stage problems is not reached within the alloted CPU time, then additional Lagrangean relaxation of the BOM constraints is performed. This yields a set of second-stage subproblems whose sum of optimal solutions provides a lower bound to the optimal solution of the first-stage subproblem that did not solve to optimality. In general, the sum of the optimal solutions of the successor nodes (subproblems) of the partitioning graph shown in Figure 5 provides a lower bound for the parent subproblem (node) because the successor subproblems are obtained through Lagrangean relaxation of one or more constraints of the parent subproblem. This implies that the sum of the optimal solutions of the end nodes (nodes with no successors) in the partitioning graph (see Figure 5) provides a lower bound to the optimal solution of the root node corresponding to problem MP.

3.3. Upper Bounding Procedure. Any feasible solution of MP provides an upper bound for the optimal solution. The lower bounding HLR procedure also provides information for constructing good upper bounds for the original problem. The basic idea is to impart part of the solution structure obtained from the lower bounding problems to the upper bounding problem UBMP. This is accomplished by postulating as many setups as possible, implied by the solution of the LRMP problems, on the MP formulation. Specifically, the original MP formulation is solved with the additional restriction that the binary variables which were equal to 1 in the lower bounding solution are forced to remain equal to 1. The remaining binary variables are left free. This defines the UBMP formulation. If UBMP is feasible, then an upper bound for MP is obtained. Otherwise, UBMP is infeasible, and some of the prepostulated setups must be removed to restore feasibility. There exist three constraint sets that may have been relaxed in the LRMP problems and thus could render formulation UBMP infeasible after fixing some of the setups. These are the capacity competition constraint (eq 7), the BOM constraint (eq 2), and the capacity constraint (eq 8). The BOM constraint defines consumption variables C_{ist} whose value may change to always restore feasibility. Also, the capacity constraint is implied by the capacity competition constraint. This means that the capacity competition constraint (eq 7) is the only one that could render formulation UBMP infeasible after prepostulating some setups.

Forcing a setup binary variable Y_{fjst} to be equal to 1 is equivalent with postulating that the corresponding run length FRL_{fjst} must be positive and thus greater than MRL_{fjs}. This means that the capacity competition constraint $\sum_f FRL_{fjst} \leq H_{jst}$ will be violated if too many setups are prepostulated and the maximum available capacity is exceeded. This implies that some forced setups have to be eliminated to restore feasibility of the capacity competition constraints. Forced setups whose "unfixing" carries the highest chance of restoring feasibility thus need to be identified. The differences

$$FRL_{fjst}^{LRMP,sol} - MRL_{fjs}$$

where $FRL_{fjst}^{LRMP,sol}$ is the solution from the LRMP formulation, measure by how much each run length FRL_{fjst} may be reduced to restore feasibility of the capacity competition constraint without violating the minimum run length bound. Clearly, the run length with the smallest difference is the one with the least leverage for restoring feasibility and thus it is the best candidate to "unfix" in the UBMP formulation. This calls for an iterative procedure where additional setups are unfixed until feasibility is restored for UBMP. It is straightforward to show that only a finite number of them is needed to guarantee feasibility. In practice, only a few are needed. This iterative upper bounding procedure is summarized as follows:

Step 1: Set $Y_{fjst} = 1$ for the UBMP model if Y_{fjst} is equal to 1 in the optimal solution of the LRMP model.

Step 2: Solve UBMP. If it is feasible, terminate. Otherwise, continue with step 3.

Step 3: Identify f which minimizes

$$FRL_{fjst}^{LRMP,sol} - MRL_{fjs}$$

Free corresponding Y_{fjst} variable in formulation UBMP. Exclude current f from subsequent consideration. Go to Step 2.

3.4. Algorithmic Procedure. The algorithmic steps of the HLR procedure incorporating the hierarchical constraint relaxation scheme and the heuristic upper bound generation are as follows:

Step 1: Lagrange multiplier initialization.
  Solve LP relaxation of MP.
  Set Lagrange multipliers of capacity competition constraints to the optimal LP relaxation values.
Step 2: Lower bound generation.
  Relax capacity competition constraints.
  Decompose LRMP into M stage 1 subproblems $\mathrm{SP}_m^{st=1}$ ($m = 1, ..., M$).
  For $m = 1-M$, **do**
    Solve $\mathrm{SP}_m^{st=1}$.
    If ($\mathrm{SP}_m^{st=1}$ solves to optimality in the specified computational time)
    then
      $SP1_m^* \leftarrow$ optimal value
    else
      Solve LP relaxation of $\mathrm{SP}_m^{st=1}$.
      Identify BOM constraints to be relaxed using LP Lagrange multiplier information.
      Relax appropriate BOM constraints.
      Decompose resulting problem into N_m stage 2 subproblems $\mathrm{SP}_{m,n}^{st=2}$ ($n = 1, ..., N_m$).
      For $n = 1-N_m$, **do**
        Solve $\mathrm{SP}_{m,n}^{st=2}$.
        If ($\mathrm{SP}_{m,n}^{st=2}$ solves to optimality in the specified computational time)
        then
          $SP2_{m,n}^* \leftarrow$ optimal value
        else
          Solve LP relaxation of $\mathrm{SP}_{m,n}^{st=2}$.
          Identify capacity constraints to be relaxed using LP Lagrange multiplier information.
          Relax appropriate capacity constraints to get problem $\mathrm{SP}_{m,n}^{st=3}$.
          Solve $\mathrm{SP}_{m,n}^{st=3}$.
          Set $SP3_{m,n}^* \leftarrow$ optimal value
          $SP2_{m,n}^* \leftarrow SP3_{m,n}^*$
      end loop over n.
    $SP1_m^* \leftarrow \sum_{n=1}^{N_m} SP2_{m,n}^*$
  end loop over m.
    $\mathrm{LBD} \leftarrow \sum_{m=1}^{M} SP1_m^*$
Step 3: Employ upper bounding procedure for generating upper bound (UBD).
Step 4: Update Lagrange multipliers.
  If (subgradient updating ceases to provide improvement)
  then
    end.
Step 5: Convergence check.
  If ($\mathrm{UBD} - \mathrm{LBD} \leq \epsilon$)
  then
    end.
  else
    go to step 2.

4. Example 1

The first example is a midterm planning problem posed by McDonald and Karimi.[35] It involves a total of

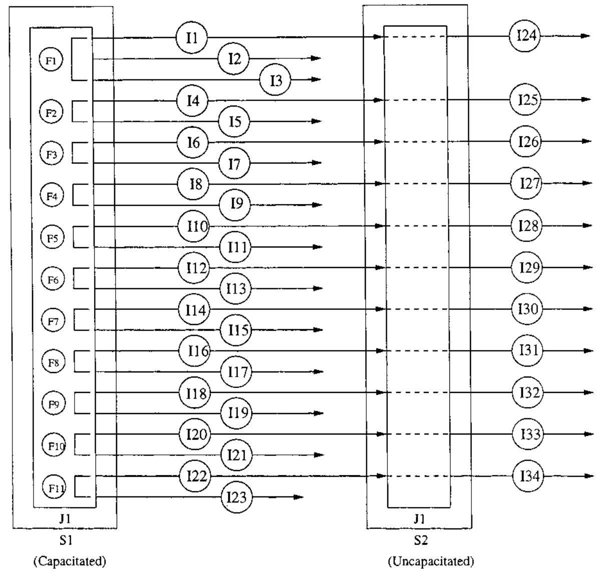

Figure 6. Serial product structure of example 1.

34 products (I_1-I_{34}) produced on two consecutive manufacturing sites involving single processors (Figure 6) and shipped to a single customer. The first facility (S_1) is capacitated, and it manufactures 23 products which are grouped into 11 product families. Some of these products are used as intermediates for producing the remaining 11 products in the second uncapacitated production site (S_2). The planning horizon consists of 12 time periods of 1 month duration with demands due for a single customer at the end of each period. The details of the supply chain and the cost parameter values can be found in McDonald and Karimi.[35] This problem was solved using OSL accessed via GAMS.[50] The minimum total cost found is $13 454. The LP relaxation involves a value of $13 203, and the relative gap between the best integer solution and the tightest lower bound is 1.6% after 100 CPU s. Interestingly, CPLEX 4.0 is much less effective, taking 1000 s to reduce the relative gap to 3.2%. As mentioned earlier, the application of Benders decomposition on this problem does not improve on the performance of commercial MILP solvers. After the binary setup variables (Y_{fjst}) are chosen as the complicating variables, upper and lower bounds of $18 054 and $13 203, respectively, are obtained in 30 iterations of the Benders decomposition procedure. The poor quality of these bounds can be attributed, as in most applications of Benders decomposition, to the low quality of the Benders cuts added to the master problem. Because an exponential number of such cuts is needed to generate the exact master, even after a large number of iterations, the lower bound fails to move above the LP relaxation value of $13 203. Variations on the standard Benders decomposition, involving suboptimal solution of the master problem and different choices of complicating variables, fail to provide better bounds.

Next, the same problem instance is solved using the proposed HLR procedure. All computations are carried out using CPLEX 4.0 interfaced through GAMS. The reported CPU times are in seconds on an IBM RS6000-397. Stage 1 of the hierarchical relaxation results in 11 disjoint first-stage subproblems, one for each product family of site S_1 and subsequent final product manufactured in site S_2 (see Figure 6). A CPU limit of 10 s is set for the solution of each subproblem. All stage 1 subproblems are solved to optimality in the specified resource limit. This obviates the need for stage 2 or 3 problem relaxation. The first iteration of the solution algorithm yields a lower bound of $13 381. The optimal production plan (Y_{fjst}, FRL_{fjst}) obtained for the first-stage subproblems is feasible for the original problem; thus, only a single pass of the upper bounding procedure is needed to generate the upper bound of $13 566. A total of 9 CPU s is needed to solve all 11 first-stage subproblems and the upper bounding problem. These bounds represent a relative gap of 1.4%, which is comparable to the 1.6% gap established in 100 CPU s by McDonald and Karimi.[35] Subsequent subgradient optimization iterations produce appreciable improvement in the bounds. These improvements cease after 17 iterations, yielding a lower bound of $13 405 and a best upper bound of $13 449 (relative gap is 0.3%) utilizing a total CPU time of 109 s. Note that this upper bound happens to be slightly better than the one found by McDonald and Karimi.[35]

Next, a variation of the previous example is considered to evaluate whether the performance of the HLR procedure depends on the product family aggregation assumption. Specifically, the aggregation of products into families in site S_1 is removed. This increases the number of possible setups and corresponding binary variables. The minimum run length and fixed cost for each individual product are calculated by dividing the values of these parameters (as given in McDonald and Karimi[35] for product families) by the number of products forming that family. For example, the first family is comprised of I_1, I_2, and I_3 with a minimum run length of 22 h and a fixed charge of $4.4. The new minimum run lengths and fixed charges for products I_1, I_2, and I_3 are thus 22/3 h and $4.4/3, respectively. This results in an increase in the number of stage 1 problems from 11 (with product family assumption) to 23 (without product family assumption).

This problem is first solved directly using CPLEX 4.0. The total cost of the plan obtained after 1000 CPU s is $14 059. The relaxed MIP has a value of $13 168. The gap between the integer solution and the best lower bound ($13 188) obtained after 1000 CPU s is 6.6%. Similar results are obtained with OSL. Clearly, the computational performance of the MILP solvers deteriorates significantly after removing the product family assumption. The first iteration of the HLR procedure yields a lower bound of $13 389 and an upper bound of $13 718 in 8 CPU s. The relative gap between these bounds after a single iteration is 2.4%, which is superior to the one obtained using MILP solvers directly and expending significant computational resources. After eight iterations of the HLR procedure and 40 s of CPU time, the relative gap is reduced to 0.5%. The best lower and upper bounds obtained are $13 389 and $13 458, respectively. Note that further subgradient optimization iterations do not provide any additional improvements on the relative gap. These results indicate that the HLR procedure is relatively insensitive to the total number of products (for the same supply chain) unlike the direct application of MILP solvers. This observation is consistent with the theoretical analysis of Bitran and Matsuo,[49] who have shown that (for capacity competition constraint relaxation) the relative gap becomes smaller as the total number of products increases. The next larger example expands on the first example requiring second- and third-stage relaxations.

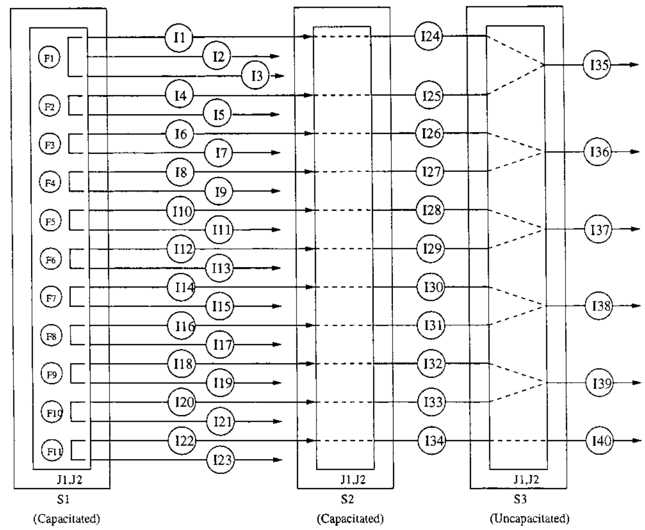

Figure 7. Assembly product structure of example 2.

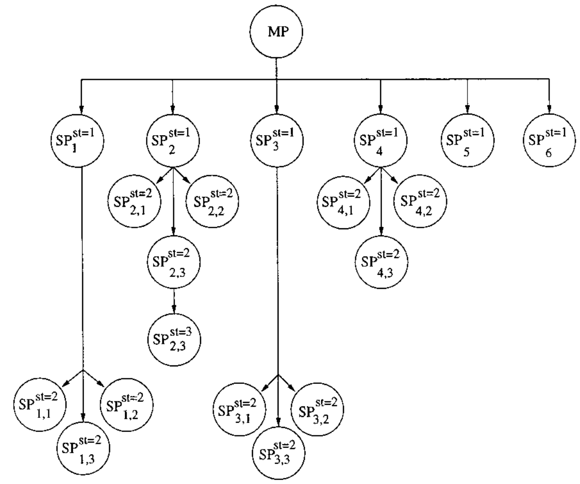

Figure 8. Tractable subproblem decomposition for example 2.

5. Example 2

The second example evaluates the effectiveness of the HLR procedure after augmenting the supply chain with a third site and an extra processor per site. It involves a total of 40 products (I_1-I_{40}), as shown in Figure 7. There are three production sites (S_1, S_2, and S_3) of which two are capacitated (S_1 and S_2) while the third (S_3) has unlimited capacity. There are two processors (J_1 and J_2) at each production site. The production structure of example 1 was *serial* because only one intermediate product was used to produce the final product at site S_2. However, the supply chain of example 2 involves an *assembly* production structure because more than one intermediate product is required for the production of products $I_{35}-I_{40}$.

This midterm planning problem is first solved directly with CPLEX 4.0. The LP relaxation has an objective function value of $19 409. After 1000 CPU s of computational time, the integer solution obtained is $23 883. The gap between this solution and the best lower bound ($19 448) is 23%. The HLR procedure generates a hierarchy of subproblems, as shown in Figure 8. The first-stage relaxation involves six subproblems with disjoint subsets of products obtained after the relaxation of the capacity competition constraints. For example, subproblem $SP_1^{st=1}$ involves product families F_1 and F_2 at site S_1, I_{24} and I_{25} at site S_2, and I_{35} at site S_3. The presence of an assembly production structure results in linking more than one families of products per site through the BOM constraints. Out of the six stage 1 subproblems, two ($SP_5^{st=1}$ and $SP_6^{st=1}$) solve to optimality in the specified resource limit of 25 CPU s. The remaining subproblems require further relaxation. On the basis of the values of the LP relaxation Lagrange multiplier, the second-stage subproblems are generated. This results in the decomposition of each of these subproblems into three stage 2 subproblems, one for each product family at site S_1 and a third for the products produced at sites S_2 and S_3. This partitioning decouples the two capacitated sites S_1 and S_2. All second-stage subproblems, except for one (i.e., $SP_{2,3}^{st=2}$), solved within the alloted CPU time of 25 s. This subproblem required relaxation of some of the capacity constraints, yielding subproblem $SP_{2,3}^{st=3}$. After eight iterations of the HLR procedure, lower and upper bounds of \$19 490 and \$21 310, respectively, are obtained. The gap between these bounds is 9.3%, and they are obtained in 376 CPU s.

6. Summary and Conclusions

In this paper, an efficient solution procedure based on Lagrangean relaxation was proposed for midterm planning problems in the process industry. The model formulation of McDonald and Karimi[35] was used as a benchmark problem for evaluating the effectiveness of the developed technique. The solution strategy consisted of bracketing the optimal solution of the original problem by decomposing it into smaller, more tractable subproblems. The lower bound was obtained by exploiting the dual structure of the model through hierarchical Lagrangean relaxation of complicating constraints. These complicating constraints were identified by a systematic procedure utilizing LP relaxation dual variable information. Horizontal and vertical partitioning of the supply chain was achieved by the proposed three-stage HLR procedure, resulting in a sequence of smaller, more tractable lower bound generating subproblems. The upper bound for the problem was generated by constructing a feasible solution through a heuristic procedure. This involved imparting part of the lower bounding solution information to the original problem. These lower and upper bound generation techniques were incorporated within a subgradient optimization procedure.

Computational results demonstrated that the proposed solution methodology was effective in bracketing the optimal value of the problem requiring relatively small computational time. Medium- to large-scale instances of the midterm planning problem, involving 30−40 products, were solved efficiently by the decomposition technique. The first example was taken from the work of McDonald and Karimi.[35] At least an order of magnitude improvement in the computational requirements was obtained over the direct application of MILP solvers. The effect of relaxing the product family assumption was then studied under the proposed solution framework. The relative advantage of the proposed technique over direct solution using branch and bound was found to be enhanced under such circumstances in accordance with previously reported results in literature. Therefore, the hierarchical decomposition technique is expected to yield better results for large-scale real-life problems involving a large number of products and setup variables. The applicability of the proposed solution technique under a more complex, assembly-type product structure was investigated in example 2. The new product structure was found to considerably reduce the solvability of the original problem using commercial solvers. This change was reflected in the proposed solution procedure in the form of larger stage 1 subproblems. Restoration of computational tractability, hence, required stages 2 and 3 of the HLR procedure.

It is important to note that while the HLR procedure typically provides good lower and upper bounds it has no mathematical guarantee of converging to the globally optimum solution. To this end, incorporation of the HLR procedure within a cutting plane or branch and bound framework is under investigation. Furthermore, the quantitative treatment of demand uncertainty is also being explored.

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